Mechanics 1: Problems with Vectors

1. Two ships P and Q are moving along straight lines with constant velocities. Initially P is at a point O and the position vector of Q relative to O is $(6\mathbf{i} + 12\mathbf{j})$ km, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors directed due east and due north respectively. The ship P is moving with velocity $10\mathbf{j}$ km h^{-1} and Q is moving with velocity $(-8\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time t hours the position vectors of P and Q relative to O are $\mathbf{p}$ km and $\mathbf{q}$ km respectively.

(a) Find $\mathbf{p}$ and $\mathbf{q}$ in terms of t.

(3)

(b) Calculate the distance of Q from P when $t = 3$.

(3)

(c) Calculate the value of t when Q is due north of P.

(2)
(Total 8 marks)

2. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}. At time t seconds, its velocity is $\mathbf{v}$ m s^{-1}. When $t = 0$, $\mathbf{v} = -2\mathbf{i} + 7\mathbf{j}$.

(a) Find the value of t when P is moving parallel to the vector $\mathbf{i}$.

(4)

(b) Find the speed of P when $t = 3$.

(3)

(c) Find the angle between the vector $\mathbf{j}$ and the direction of motion of P when $t = 3$.

(3)
(Total 10 marks)

3. A particle P moves in a horizontal plane. The acceleration of P is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-2}. At time $t = 0$, the velocity of P is $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

(a) Find, to the nearest degree, the angle between the vector $\mathbf{j}$ and the direction of motion of P when $t = 0$.

(3)

At time t seconds, the velocity of P is $\mathbf{v}$ m s^{-1}. Find

(b) an expression for $\mathbf{v}$ in terms of t, in the form $a\mathbf{i} + b\mathbf{j}$,

(2)

(c) the speed of P when $t = 3$,

(3)

(d) the time when P is moving parallel to $\mathbf{i}$.

(2)
(Total 10 marks)

4. A particle P of mass 3 kg is moving under the action of a constant force $\mathbf{F}$ newtons. At $t = 0$, P has velocity $(3\mathbf{i} - 5\mathbf{j})$ m s^{-1}. At $t = 4$ s, the velocity of P is $(-5\mathbf{i} + 11\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P, in terms of $\mathbf{i}$ and $\mathbf{j}$.

(2)

(b) the magnitude of $\mathbf{F}$.

(4)

At $t = 6$ s, P is at the point A with position vector $(6\mathbf{i} - 29\mathbf{j})$ m relative to a fixed origin O. At this instant the force $\mathbf{F}$ newtons is removed and P then moves with constant velocity. Three seconds after the force has been removed, P is at the point B.

(c) Calculate the distance of B from O.

(6)
(Total 12 marks)

5. [*In this question, the unit vectors* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of the ball,

(2)

(b) the position vector of the ball after t seconds.

(2)

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(c) Find the time when the ball is due north of B.

(2)

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(d) find the value of v.

(6)

(e) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

(1)
(Total 13 marks)

6. A small boat S, drifting in the sea, is modelled as a particle moving in a straight line at constant speed. When first sighted at 0900, S is at a point with position vector $(4\mathbf{i} - 6\mathbf{j})$ km relative to a fixed origin O, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north respectively. At 0945, S is at the point with position vector $(7\mathbf{i} - 7.5\mathbf{j})$ km. At time t hours after 0900, S is at the point with position vector $\mathbf{s}$ km.

(a) Calculate the bearing on which S is drifting.

(4)

(b) Find an expression for $\mathbf{s}$ in terms of t.

(3)

At 1000 a motor boat M leaves O and travels with constant velocity $(p\mathbf{i} + q\mathbf{j})$ km h^{-1}. Given that M intercepts S at 1015,

(c) calculate the value of p and the value of q.

(6)
(Total 13 marks)

7. [In this question, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due East and North respectively.]

A coastguard station O monitors the movements of ships in a channel. At noon, the station's radar records two ships moving with constant speed. Ship A is at the point with position vector $(-5\mathbf{i} + 10\mathbf{j})$ km relative to O and has velocity $(2\mathbf{i} + 2\mathbf{j})$ km h^{-1}. Ship B is at the point with position vector $(3\mathbf{i} + 4\mathbf{j})$ km and has velocity $(-2\mathbf{i} + 5\mathbf{j})$ km h^{-1}.

(a) Given that the two ships maintain these velocities, show that they collide.

(6)

The coast guard radios ship A and orders it to reduce its speed to move with velocity $(\mathbf{i} + \mathbf{j})$ km h^{-1}.

Given that A obeys this order and maintains this new constant velocity,

(b) find an expression for the vector $\overrightarrow{AB}$ at time t hours after noon.

(2)

(c) find, to 3 significant figures, the distance between A and B at 1400 hours,

(3)

(d) find the time at which B will be due north of A.

(2)

(Total 13 marks)

8. [*In this question the vectors* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors in the directions due east and due north respectively*.]

Two boats A and B are moving with constant velocities. Boat A moves with velocity $9\mathbf{j}$ km h^{-1}. Boat B moves with velocity $(3\mathbf{i} + 5\mathbf{j})$ km h^{-1}.

(a) Find the bearing on which B is moving.

(2)

At noon, A is at point O, and B is 10 km due west of O. At time t hours after noon, the position vectors of A and B relative to O are $\mathbf{a}$ km and $\mathbf{b}$ km respectively.

(b) Find expressions for $\mathbf{a}$ and $\mathbf{b}$ in terms of t, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$.

(3)

(c) Find the time when B is due south of A.

(2)

At time t hours after noon, the distance between A and B is d km. By finding an expression for $\overrightarrow{AB}$,

(d) show that $d^2 = 25t^2 - 60t + 100$.

(4)

At noon, the boats are 10 km apart.

(e) Find the time after noon at which the boats are again 10 km apart.

(3)

(Total 14 marks)

9. Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\mathbf{i} + 10\mathbf{j})$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(14\mathbf{i} - 6\mathbf{j})$ km. Three hours later, P is at the point with position vector

(29**i** + 34**j**) km. The ship Q travels with velocity 12**j** km h^{-1}. At time t hours after midnight, the position vectors of P and Q are **p** km and **q** km respectively. Find

(a) the velocity of P, in terms of **i** and **j**,

(2)

(b) expressions for **p** and **q**, in terms of t, **i** and **j**.

(4)

At time t hours after midnight, the distance between P and Q is d km.

(c) By finding an expression for $\overrightarrow{PQ}$, show that

$$d^2 = 25t^2 - 92t + 292.$$

(5)

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into sight of the observer,

(d) find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

(5)

(Total 16 marks)