Mechanics 1: Answers to Problems with Vectors

1. (a) $\mathbf{p} = 10t\mathbf{j}$ B1

$\mathbf{q} = (6\mathbf{i} + 12\mathbf{j}) + (-8\mathbf{i} + 6\mathbf{j})t$ M1 A1 3

(b) $t = 3$: $\mathbf{p} = 30\mathbf{j}$, $\mathbf{q} = -18\mathbf{i} + 30\mathbf{j}$ M1 A1

$\Rightarrow$ dist. apart = 18 km A1 3

Alt. (b)

$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = (6 - 8t)\mathbf{i} + (12 - 4t)\mathbf{j}$ M1

$t = 3$: $\mathbf{PQ} = -18\mathbf{i} + 0\mathbf{j}$ | or $|\mathbf{PQ}|^2 = (6 - 8t)^2 + (12 - 4t)^2$ A1

Dist. = 18 km | $t = 3 \rightarrow |\mathbf{PQ}| = 18$ A1

(c) Q north of $P \Rightarrow 6 - 8t = 0$ M1

$t = \frac{3}{4}$ A1 2

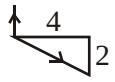
[8]

2. (a) "$v = u + at$" : $\mathrm{v} = (-2 + 2t)\mathrm{i} + (7 - 3t)\mathrm{j}$ M1 A1

v parallel to i $\Rightarrow 7 - 3t = 0 \Rightarrow t = 2\frac{1}{3}$ s M1 A1 4

(b) $t = 3$, $\mathrm{v} = 4\mathrm{i} - 2\mathrm{j}$ M1

$|\mathrm{v}| = \sqrt{20} \approx \underline{4.47}$ m s^{-1} M1 A1 3

(c) 

Angle = (arctan $\frac{2}{4}$), + 90° = 116.6° (accept 117°) M1, M1 A1 3

[or 180° – (arctan $\frac{4}{2}$)] [M1 M1 A1]

[10]

3. (a) $\tan\theta = \frac{3}{2}$ ($\theta = 56.3°$) M1

angle between $\mathbf{v}$ and $\mathbf{j}$ = 90 + 56.3 ≈ 146° M1 A1 3

(b) $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + (-\mathbf{i} + 2\mathbf{j})t$ M1

$= (2 - t)\mathbf{i} + (-3 + 2t)\mathbf{j}$ A1 2

(c) $t = 3$, $\mathbf{v} = -\mathbf{i} + 3\mathbf{j}$ M1

speed = $\sqrt{(1^2 + 3^2)} = \underline{\sqrt{10 \text{ or } 3.16 \text{ m s}^{-1}}}$ M1 A1 3

(d) $\mathbf{v}$ parallel to $\mathbf{i} \Rightarrow -3 + 2t = 0$ M1

$\Rightarrow t = \underline{1.5 \text{ s}}$ A1 2

[10]

4. (a) $a = \frac{1}{4}[(5\mathbf{i} + 11\mathbf{j}) - (3\mathbf{i} - 5\mathbf{j})] = -2\mathbf{i} + 4\mathbf{j}$ M1 A1 2

(b) $\mathbf{F} = m\mathbf{a} = -6\mathbf{i} + 12\mathbf{j}$ M1 A1

$|\mathbf{F}| = \sqrt{180} \cong 13.4$ N (AWRT) M1 A1 4

[**OR** $|\mathbf{a}| = \sqrt{20} \cong 4.47 \Rightarrow |\mathbf{F}| = 3 \times 4.47 \cong 13.4$ N]

(c) $t = 6$ $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j} + 6(-2\mathbf{i} + 4\mathbf{j})$ [$= -9\mathbf{i} + 19\mathbf{j}$] M1 A1

At B: $\mathbf{r} = (6\mathbf{i} - 29\mathbf{j}) + 3(-9\mathbf{i} + 19\mathbf{j})$ [$= -21\mathbf{i} + 28\mathbf{j}$] M1 A1ft

OB = $\sqrt{(21^2 + 28^2)}$ = <u>35 m</u> M1 A1 6

[12]

5. (a) Speed of ball = $\sqrt{(5^2 + 8^2)} \approx$ <u>9.43 m s^{-1}</u> M1 A1 2

M1 Valid attempt at speed (square, add and squ. root cpts)

(b) p.v. of ball = (2**i** + **j**) + (5**i** + 8**j**)*t* M1 A1 2

M1 needs non-zero p.v. + (attempt at veloc vector) x t. Must be vector

(c) North of *B* when **i** components same, i.e. 2 + 5*t* = 10 M1

t = <u>1.6 s</u> A1 2

(d) When *t* = 1.6, p.v. of ball = 10**i** + 13.8**j** (or **j** component = 13.8) M1A1

↓

Distance travelled by 2nd player = 13.8 – 6 = 6.8 M1A1

↓

Speed = 6.8 ÷ 1.6 = <u>4.25 m s^{-1}</u> M1A1 6

or [(2 + 5*t*)**i** +] (1 + 8*t*)**j** = [10**i** +] (7 + *vt*)**j** M1A1

*(pv's or **j** components same)*

↓

Using *t* = 1.6: 1 + 12.8 = 7 + 1.6*v* (equn in *v* only) M1A1

↓

v = <u>4.25 m s^{-1}</u> M1A1

2nd M1 – allow if finding displacement vector (e.g. if using wrong time)

3rd M1 for getting speed as a scalar (and final answer must be as a scalar). But if they get e.g. '4.25j', allow M1 A0

(e) Allow for friction on field (i.e. velocity of ball not constant) B1 1

or allow for vertical component of motion of ball

Allow 'wind', 'spin', 'time for player to accelerate', size of ball

Do not allow on their own 'swerve', 'weight of ball'.

[13]

6. (a) Direction of **v** = (7**i** – 7.5**j**) – (4**i** – 6**j**) = 3**i** – 1.5**j** M1

↓

tan *q* = $\frac{1.5}{3}$ = 0.5 ⇒ *q* = 26.565… M1 A1

Bearing = <u>117</u> (accept awrt) A1 4

(b) **v** = (3**i** – 1.5**j**) ÷ $\frac{3}{4}$ = 4**i** – 2**j** B1

s = <u>(4**i** – 6**j**) + *t*(4**i** – 2**j**)</u> M1 A1f.t. 3

(c) At 1015 **s** = (4**i** – 6**j**) + $\frac{5}{4}$(4**i** – 2**j**) (= 9**i** – 8.5**j**) M1 A1

m = 0.25 (*p***i** + *q***j**) B1

↓

s = **m** ⇒ <u>*p* = 36, *q* = – 34</u> M1 A1, A1 6

[13]

7. (a) At time *t* $\mathbf{r}_A$ = (–5 + 2*t*)**i** + (10 + 2*t*)**j** B1

$\mathbf{r}_B = (3 - 2t)\mathbf{i} + (4 + 5t)\mathbf{j}$ B1

i components equal when $-5 + 2t = 3 - 2t \Rightarrow t = 2$ h M1 A1

$t = 2$: $\mathbf{r}_A = -\mathbf{i} + 14\mathbf{j}$; $\mathbf{r}_B = -\mathbf{i} + 14\,\mathbf{j} \Rightarrow$ collide M1 A1 6

(b) New $\mathbf{r}_A = (-5 + t)\mathbf{i} + (10 + t)\,\mathbf{j}$

(c) $\Rightarrow AB = \mathbf{r}_B - \mathbf{r}_A = (8 - 3t)\mathbf{i} + (-6 + 4t)\mathbf{j}$ M1 A1 2

$t = 2$: $\overrightarrow{AB} = 2\mathbf{i} + 2\mathbf{j}$, $\Rightarrow$ dist. $= \sqrt{(2^2 + 2^2)} \approx 2.83$ km M1 M1 A1 3

(d) B north of $A \Rightarrow 8 - 3t = 0 \Rightarrow t = 8/3 \Rightarrow$ time 1440 hours

[11]

8. (a) $\tan\theta = \frac{3}{5} \Rightarrow \theta = 031°$ M1 A1 2

(b) $\mathbf{a} = 9t\,\mathbf{j}$ B1

$\mathbf{b} = (-10 + 3t)\mathbf{I} + 5t\,\mathbf{j}$ M1 A1 3

(c) B south of A $\Rightarrow -10 + 3t = 0$ M1

$t = 3\frac{1}{3} \Rightarrow$ <u>1520 hours</u> A1 2

(d) $AB = \mathbf{b} - \mathbf{a} = (3t - 10)\mathbf{I} + 5t\,\mathbf{i}$ M1 A1

↓

$d^2 = |\mathbf{b} - \mathbf{a}|^2 = (3t - 10)^2 + 16t^2$ M1

$= 25t^2 - 60t + 100$ (*) A1 4

(e) $d = 10 \Rightarrow d^2 = 100 \Rightarrow 25t^2 - 60t = 0$ M1

$\Rightarrow t = (0$ or 2.4 A1

$\Rightarrow$ time <u>1424 hours</u> A1 3

[15]

9. (a) $\mathbf{v}_P = \{(29\mathbf{i} + 34\mathbf{j}) - (20\mathbf{i} + 10\mathbf{j})\}/3 = \underline{(3\mathbf{i} + 8\mathbf{j})\text{ km h}^{-1}}$ M1 A1 2

(b) $\mathbf{p} = (20\mathbf{i} + 10\mathbf{j}) + (3\mathbf{i} + 8\mathbf{j})t$ M1 A1ft

$\mathbf{q} = (14\mathbf{i} - 6\mathbf{j}) + 12t\mathbf{j}$ M1 A1 4

(c) $\mathbf{q} - \mathbf{p} = (-6 - 3t)\mathbf{i} + (-16 + 4t)\mathbf{j}$ M1 A1

↓

$d^2 = (-6 - 3t)^2 + (-16 + 4t)^2$ M1

↓

$= 36 + 36t + 9t^2 + 16t^2 - 128t + 256$ M1

$= 25t^2 - 92t + 292$ (*) A1 (cso) 5

(d) $25t^2 - 92t + 292 = 225$ M1

$25t^2 - 92t + 67 = 0$ A1

↓

$(t - 1)(25t - 67) = 0$ M1

$t = 67/25$ or 2.68 A1

time $\approx$ 161 mins, or 2 hrs 41 mins, or 2.41 am, or 0241 A1 5

[16]